CS151 Intro to Data Structures

Trees

Announcements

- **HW04 Released**
- Lab today will be part of your HW04 grade
- Exam next week!
  - I will post midterm review slides early

Outline

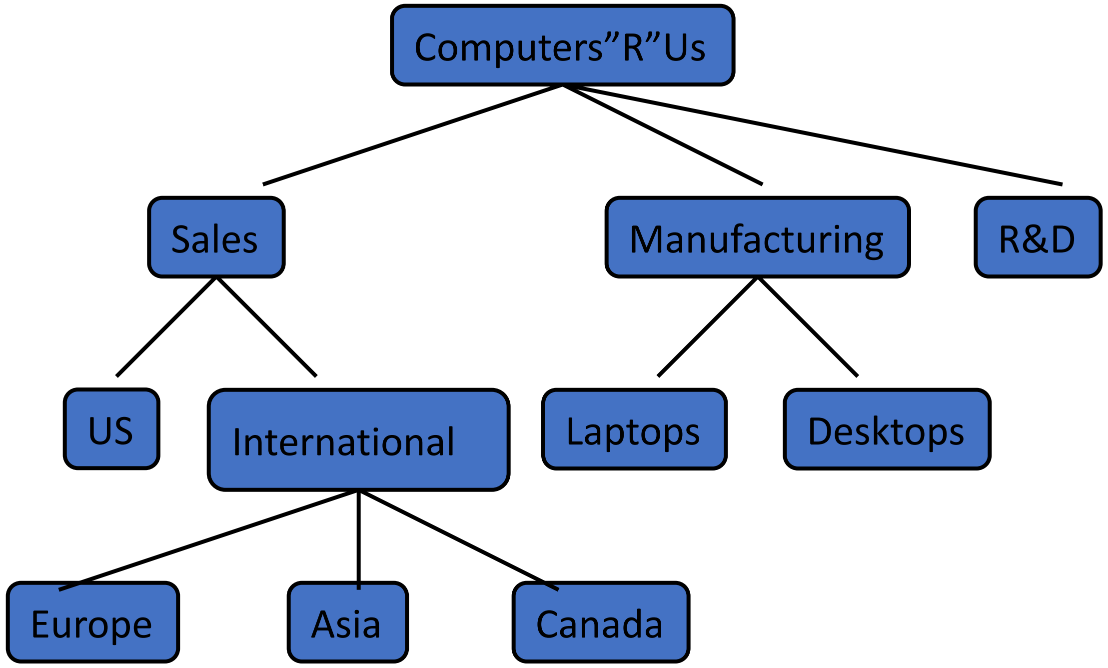
- Trees:
  - Binary Trees
  - Binary Search Trees
    - Inserting
    - Searching

Tree

A tree is a **hierarchical** data structure

Node: individual elements in the tree

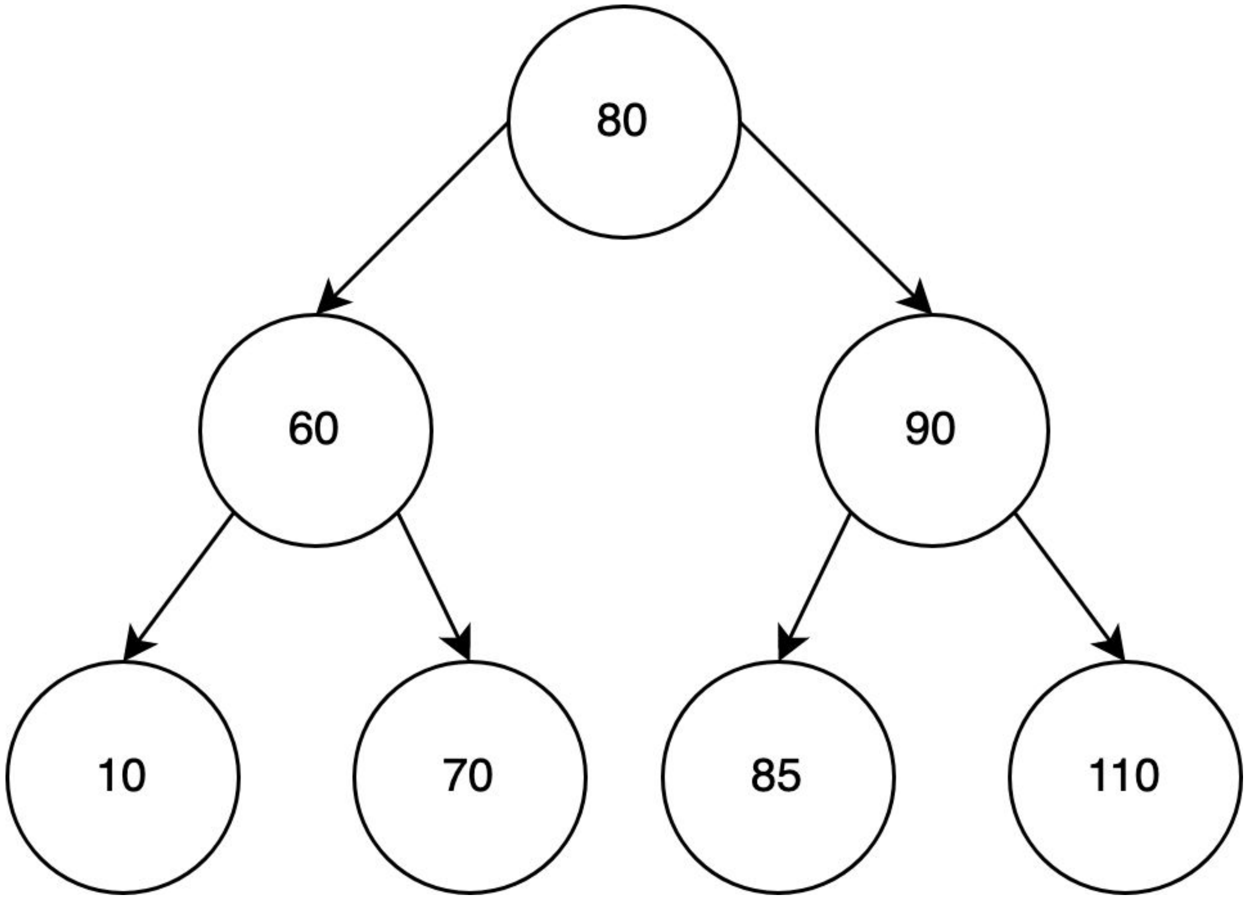
Nodes have a parent-child relation



Trees: Nodes

```
class Node {
    int key;
    Node left;
    Node right;

    public Node(int item) {
        key = item;
        left = null;
        right = null;
    }
}
```



Terminology

root: no parent

A

external/leaf node: no children

E, I, J, K, G, H, D

internal node: - node with at least one child

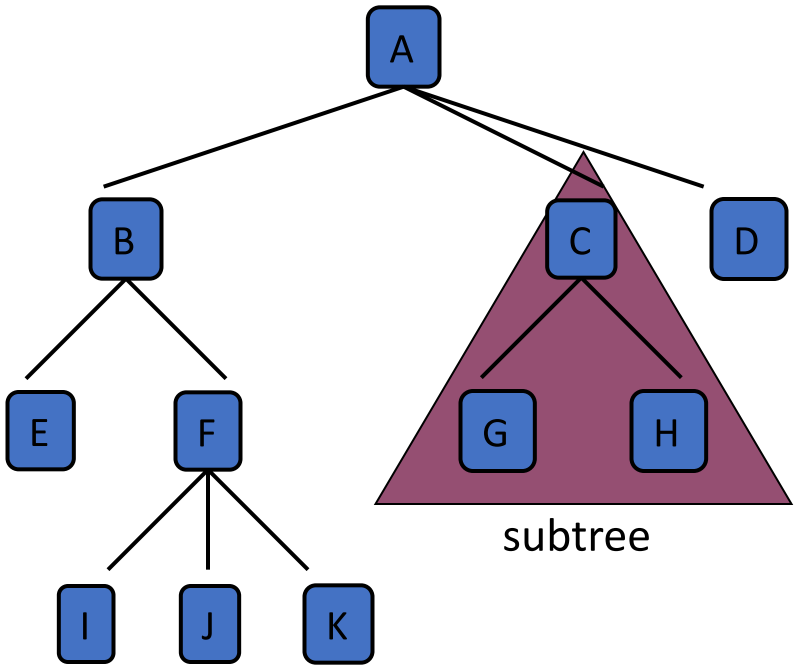
A, B, C, F

parent/child

depth - # of ancestors

Height - Maximum number of edges from a leaf node to the root

- **Subtree**: tree consisting of a node and its descendants



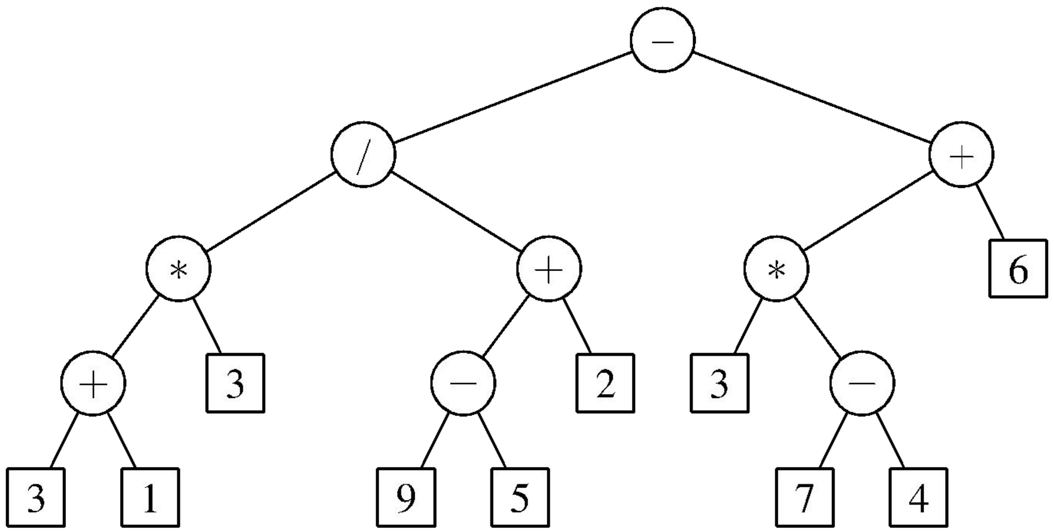
Binary Tree

Each node in a **binary tree** has at most two children

Recursive definition:

Each node has at most two children

- Both subtrees are **binary trees**



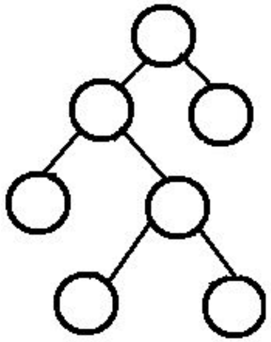
Types of Binary Trees

A binary tree is **full** (or proper) if each node has **zero or two children**

A binary tree is **complete** if every level (except possibly the last) is filled

If a complete binary tree is filled at every level, it is **perfect**

A



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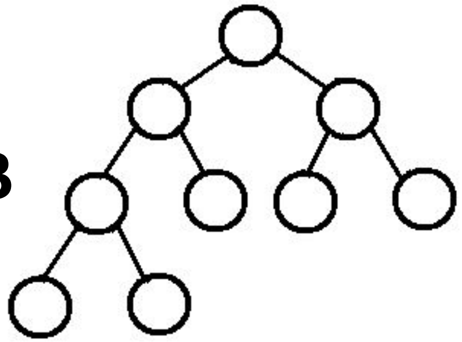

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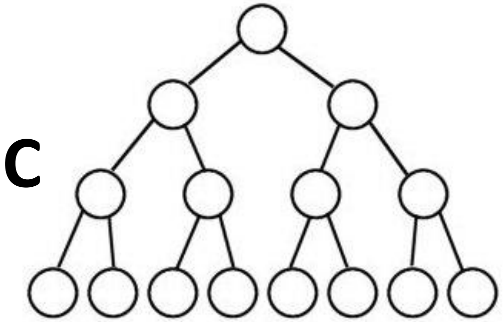


Types of Binary Trees

A binary tree is **full** (or proper) if each node has **zero or two children**

A binary tree is **complete** if every level (except possibly the last) is **full**

If a complete binary tree is filled at every level, it is **perfect**

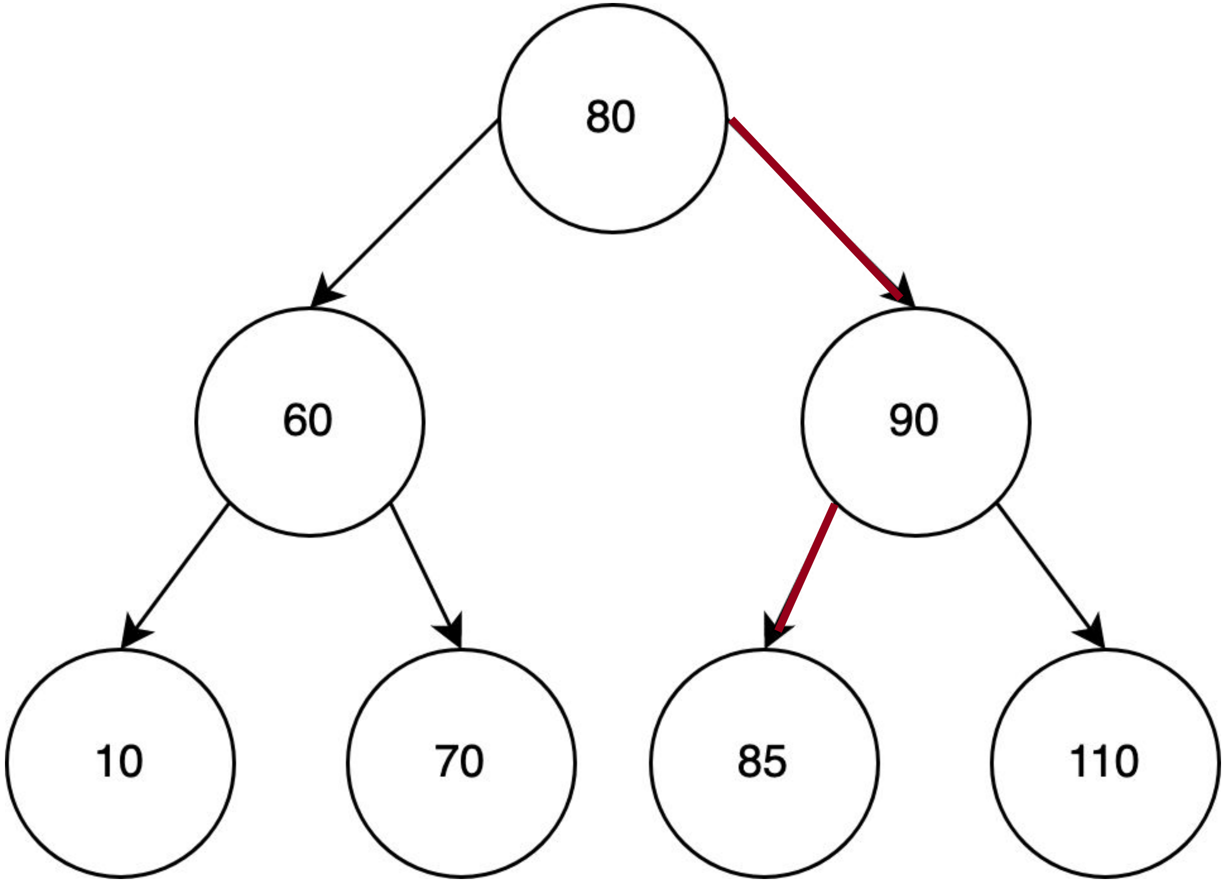
Q1: Is every full binary tree a complete binary tree?

Q2: Is every complete binary tree a full binary tree?
Q3: Is every perfect binary tree a full binary tree?

Binary Trees: Height

Height of a tree:

Maximum number of edges from a leaf node to the root



Height? 2

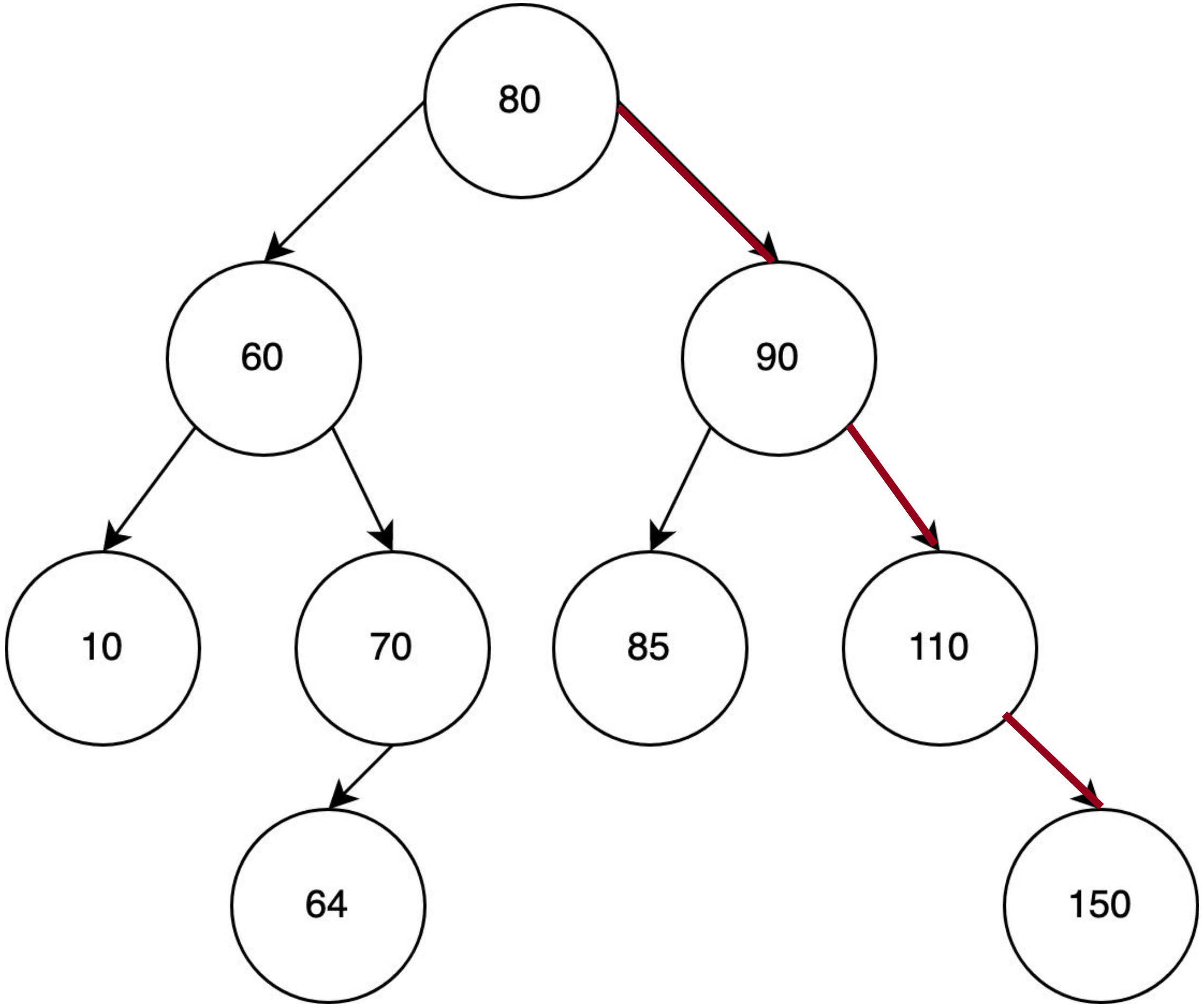
$\log_2(7) \approx 2$

Tree Review

Height? 3

$\log_2(9) \approx 3$

Height of a binary tree is roughly log(n) where n is number of nodes

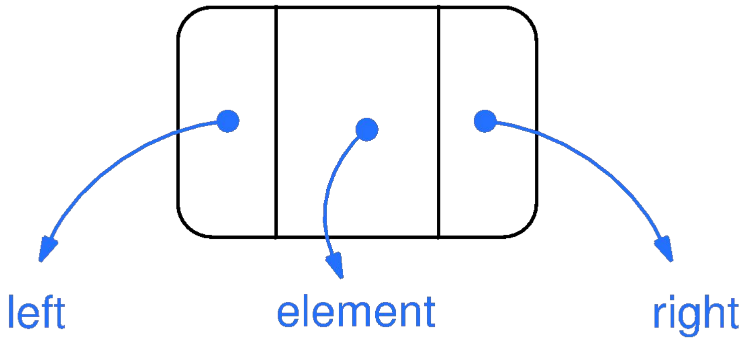


Binary Tree Interface

```
public interface BinaryTree<E extends
Comparable<E>> {
  int size();
  boolean isEmpty();
  void insert(E element);
  boolean contains(E element);
  ...
}
```

Node Implementation

```
public class Node<E> {
  private E element;
  private Node<E> left;
  private Node<E> right;
  //constructors, getters, setters
  }
}
```



Class

```
public class LinkedBinaryTree<E extends
Comparable<E>> implements BinaryTree<E> {
    // what instance variables?
    // nested Node class



}
```

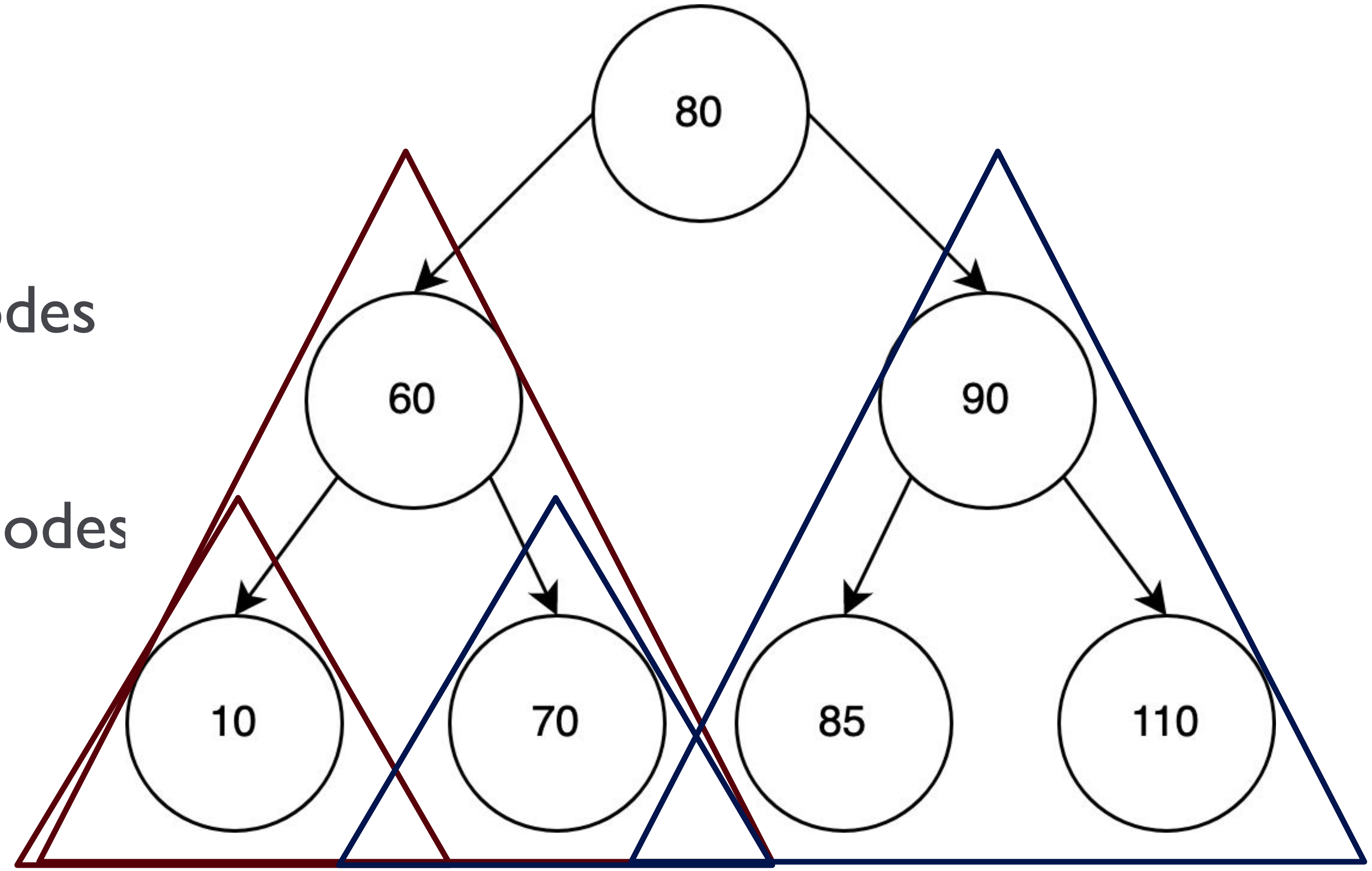
Binary Search Trees

Binary Search Trees

Definition:

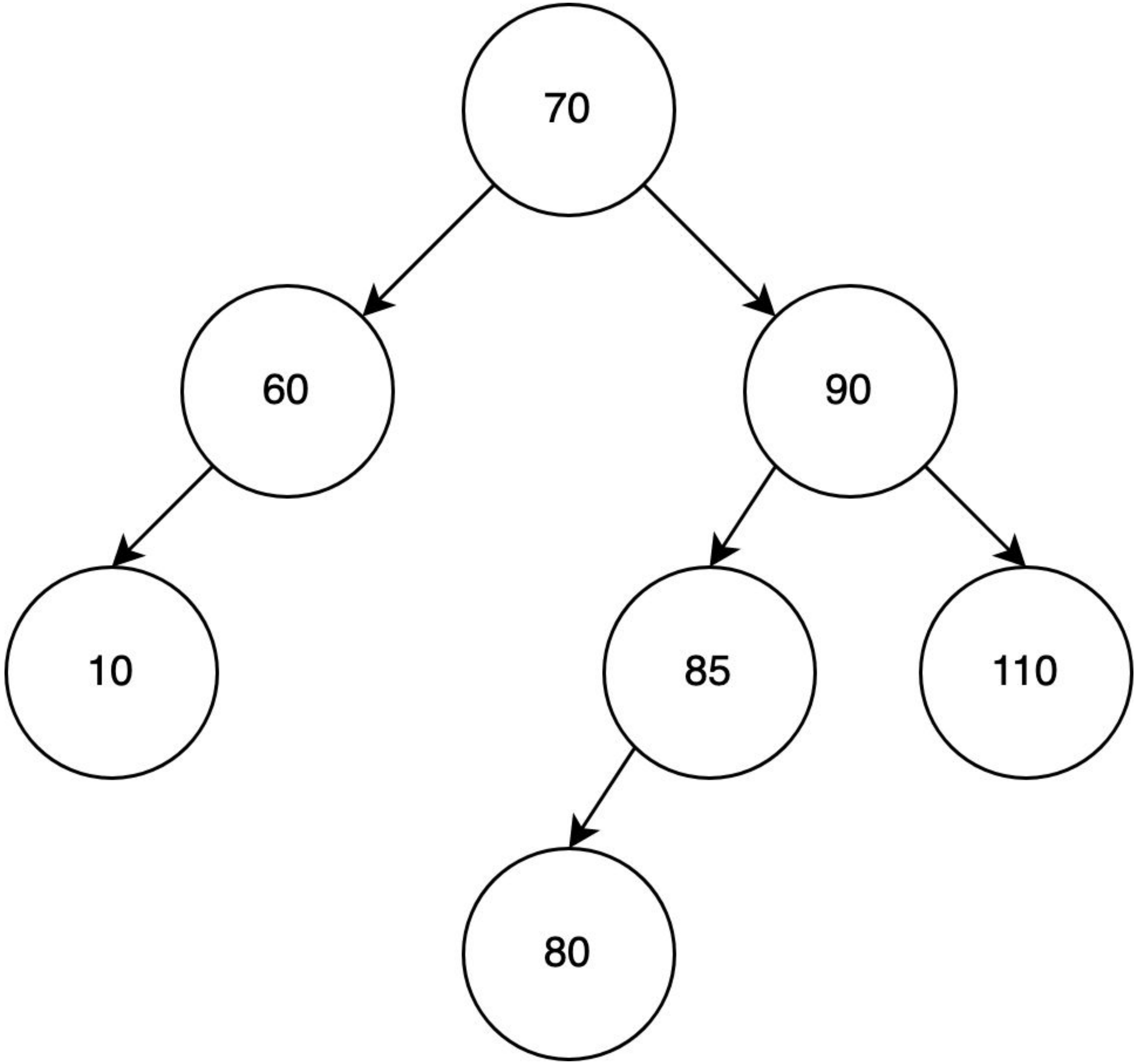
At **each node** with value **k**

- Left subtree contains only nodes with value **lesser** than **k**
- Right subtree contains only nodes with value **greater** than **k**
- Both subtrees are a **binary search tree**



Exercise One: Binary Search Trees

Is this a binary search tree?



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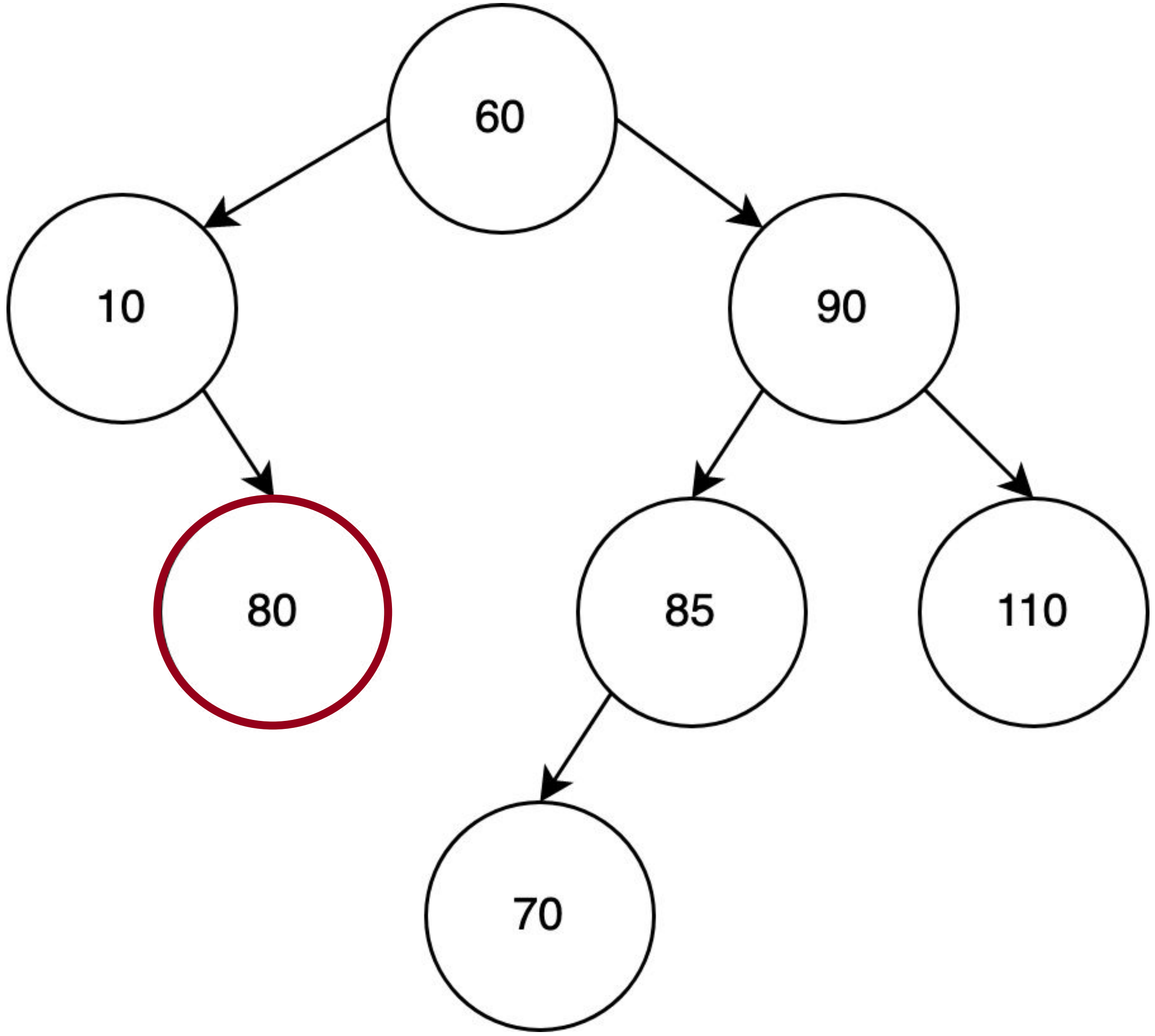

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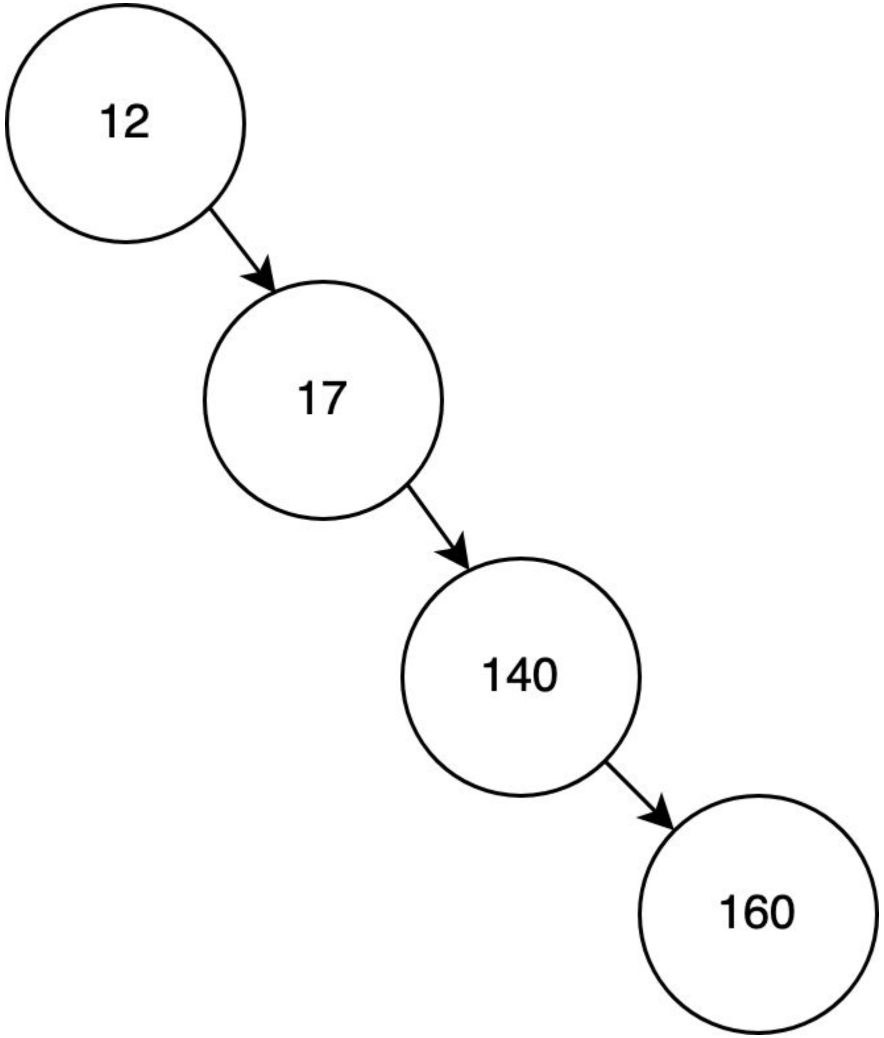
Is this a binary search tree?



Today’s Lecture

1. Binary Search Trees
2. **Search**
3. Insertion
4. Removal
5. Summary

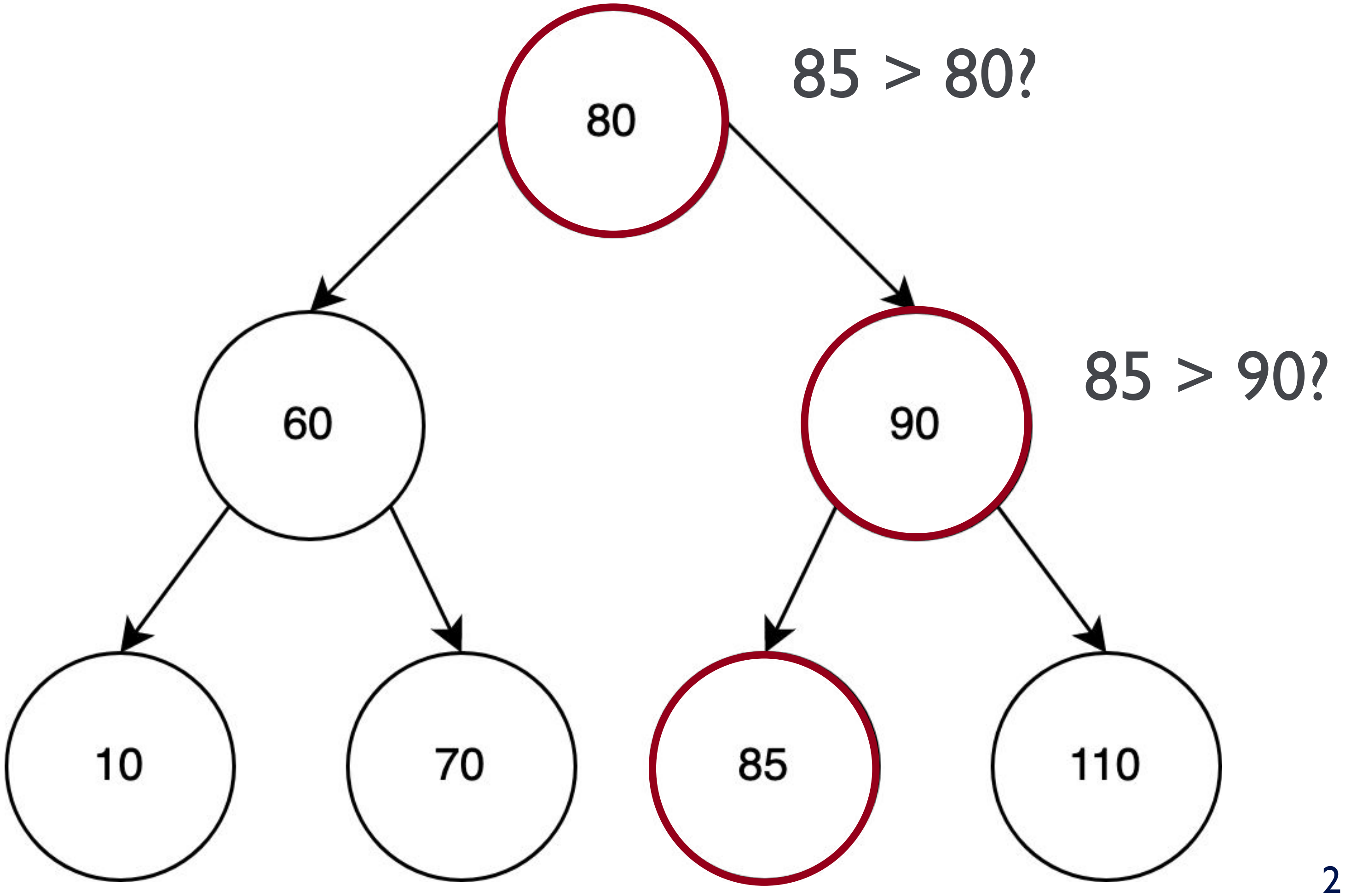
Binary Search Trees: Efficient Search

Goal: Report if a value exists in the tree

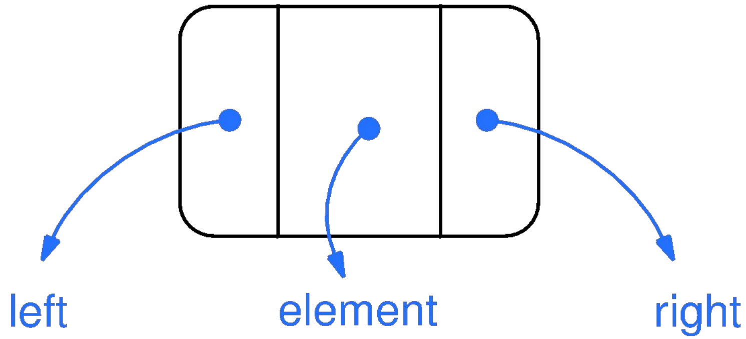
Target: 85

if **target** > **k**:
    Move right
else:
    Move Left

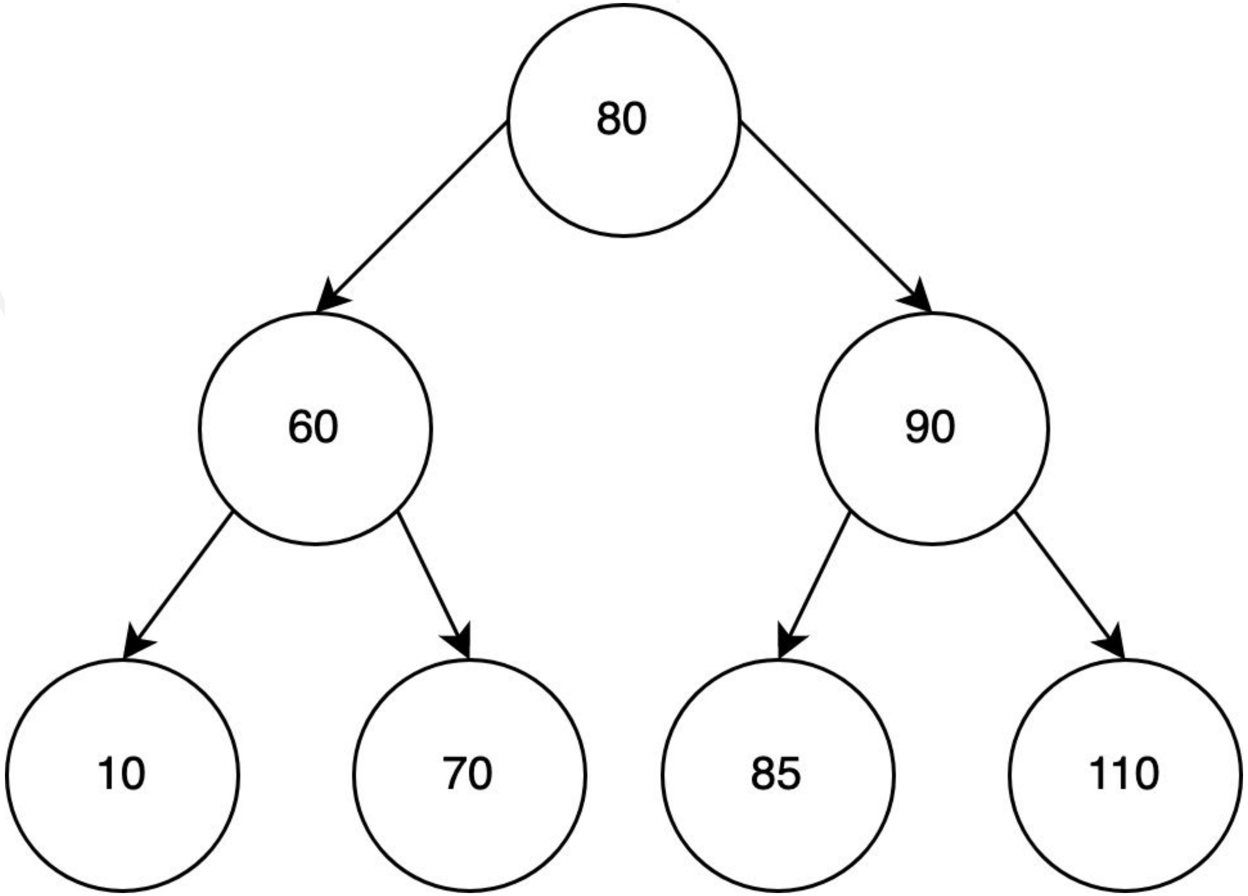
Complexity?
O(log n)



BSTs: Search Implementation



BSTs: Search Implementation



search(Node(80), 85)
    search(Node(90), 85)
        search(Node(85), 85)

Today's Lecture

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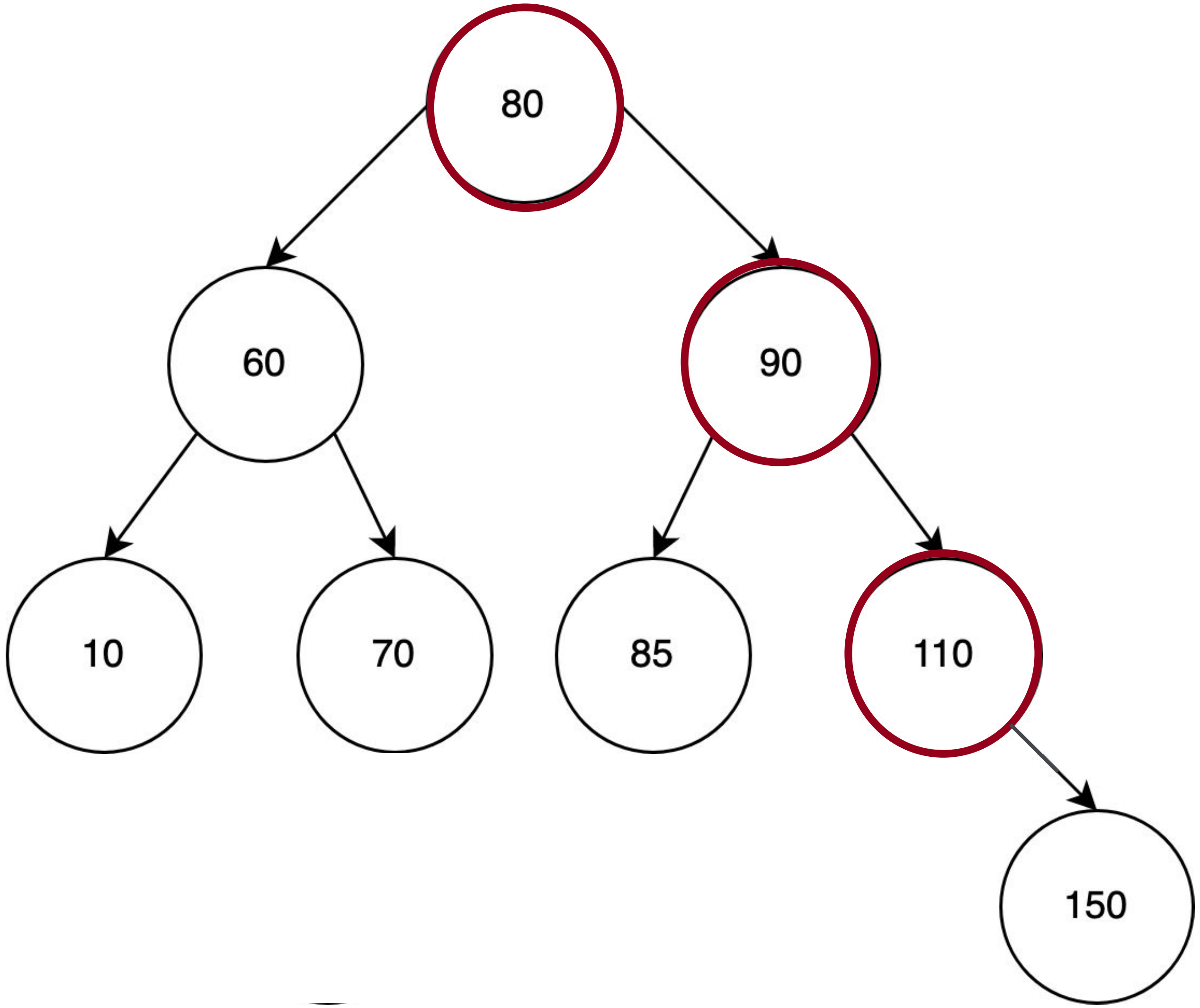
Binary Search Trees: Insertion

Insertion must maintain the properties of a BST!

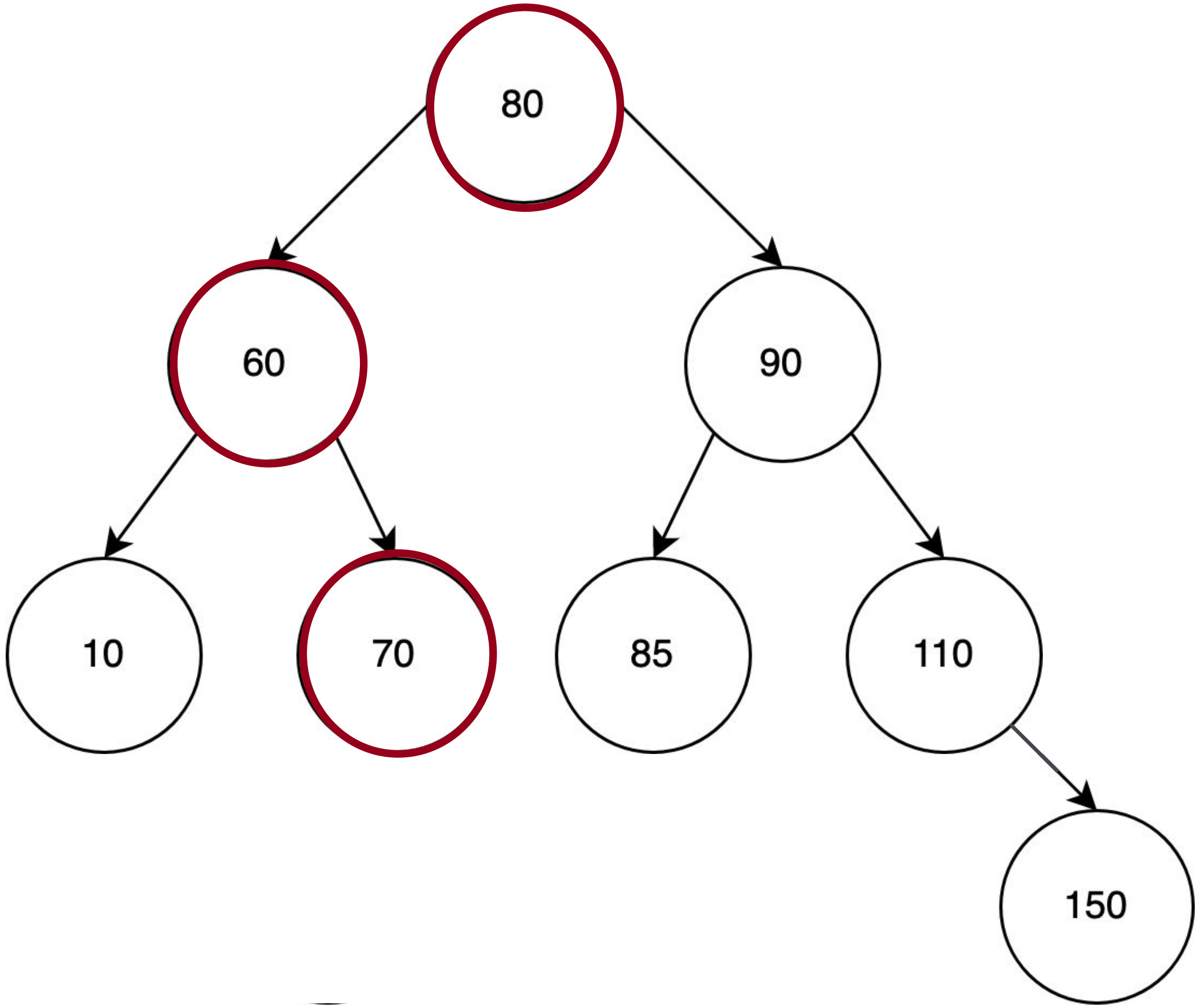
Insert: 150



Binary Search Trees: Insertion

Insertion must maintain the properties of a BST!

Insert: <u>64</u>



Complexity?
O(log n)

Today's Lecture

1. Binary Search Trees
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4. **Removal**
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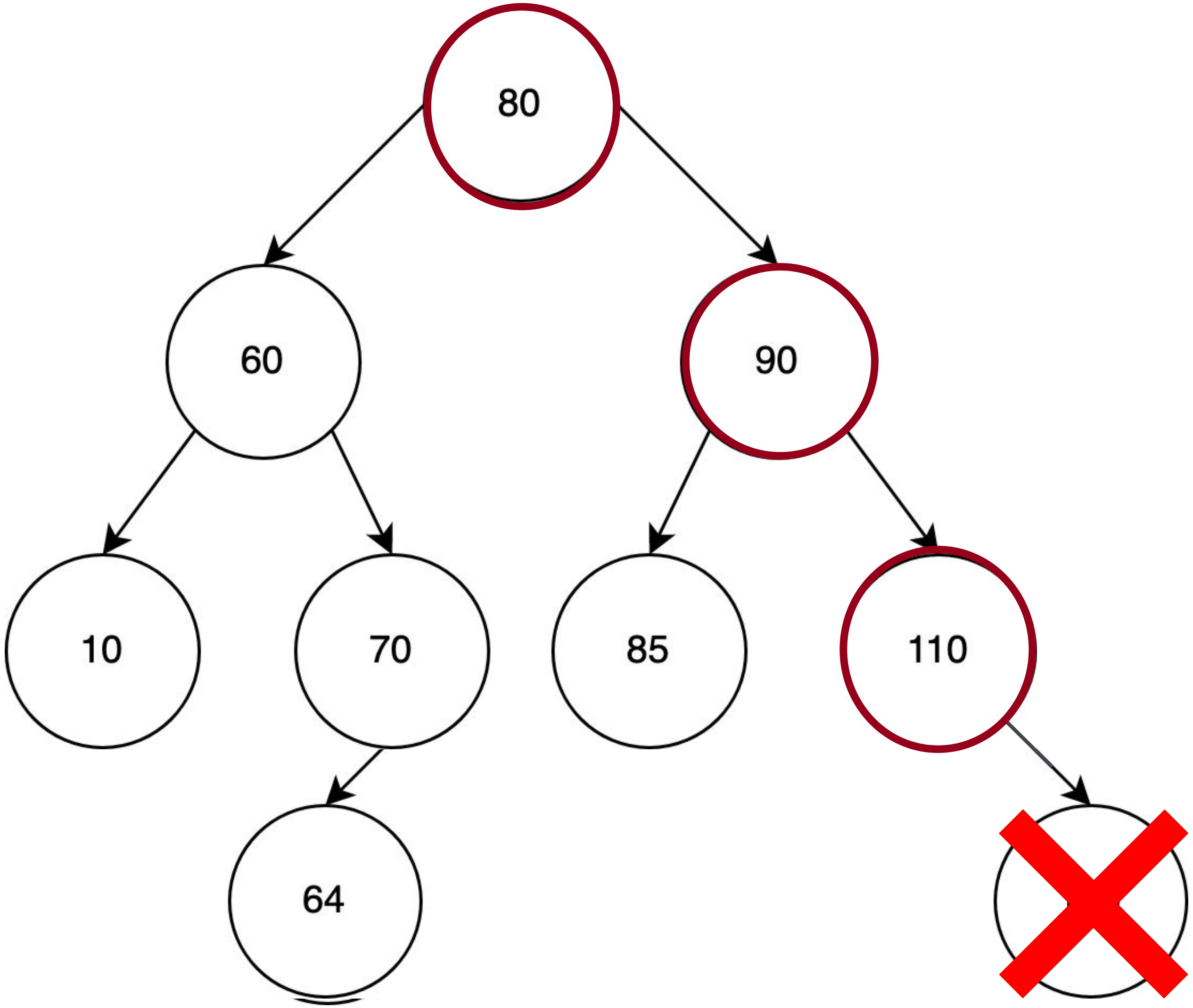
Binary Search Trees: Deletion

Deletion must maintain the properties of a BST!

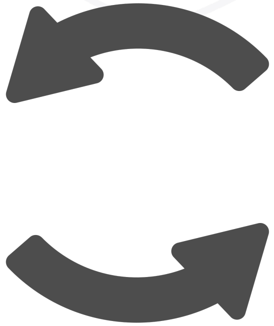
Delete: <u>150</u>

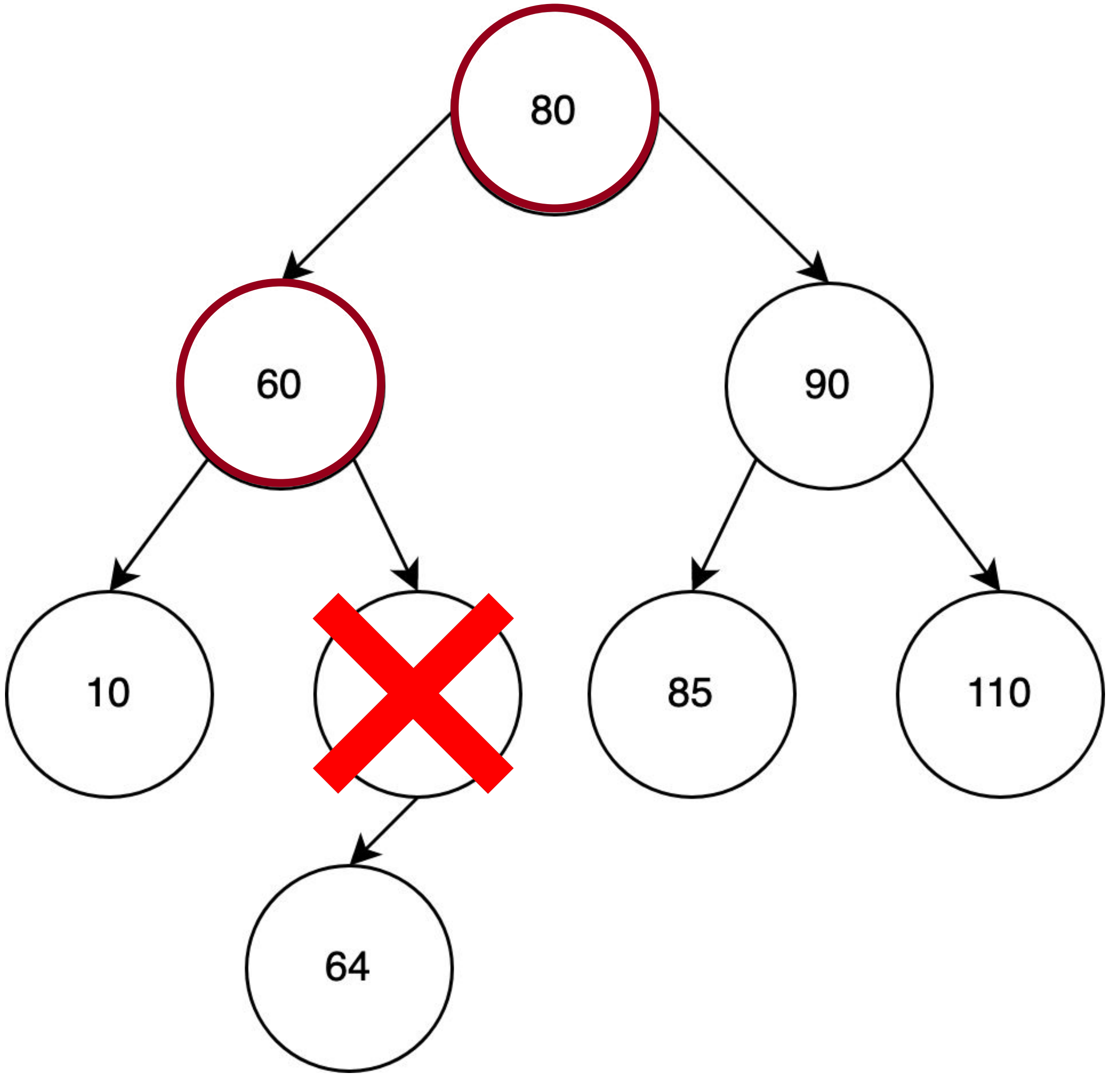


Binary Search Trees: Deletion

Deletion must maintain the properties of a BST!

Delete: 70





Binary Search Trees: Deletion

Deletion must maintain the properties of a BST!

Delete: 80

At **each node** with value **k**

- Left subtree contains only nodes with value **lesser** than **k**
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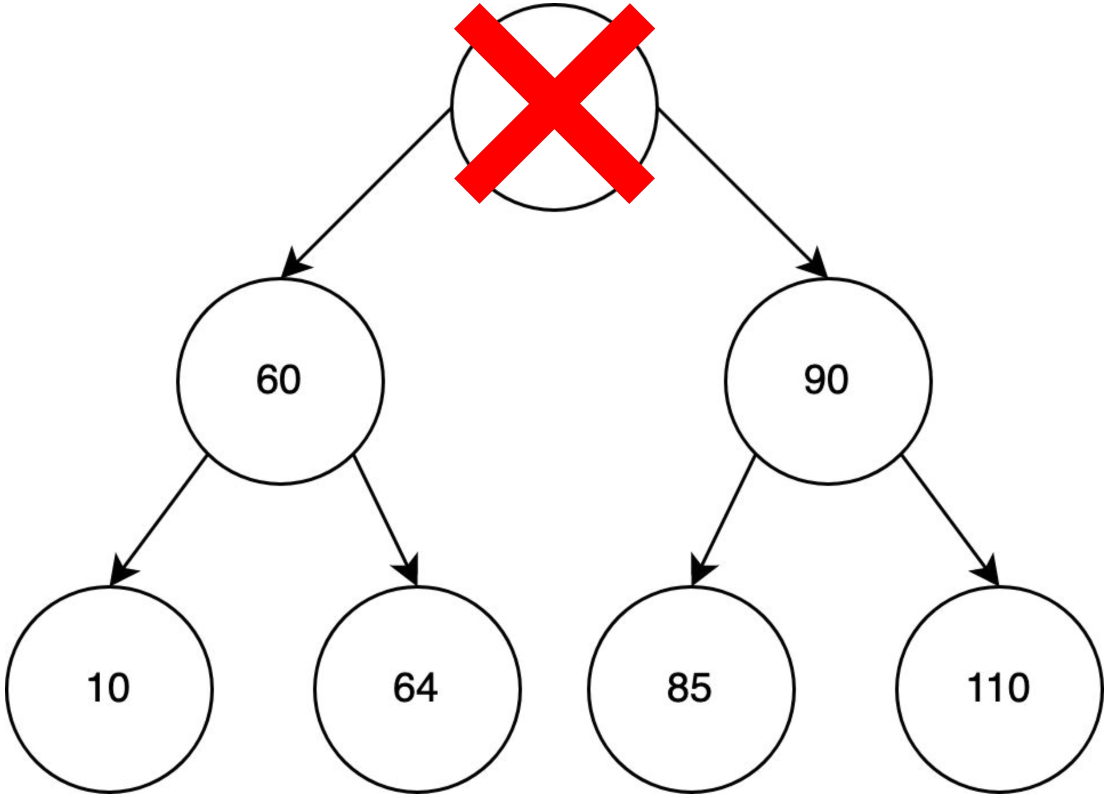
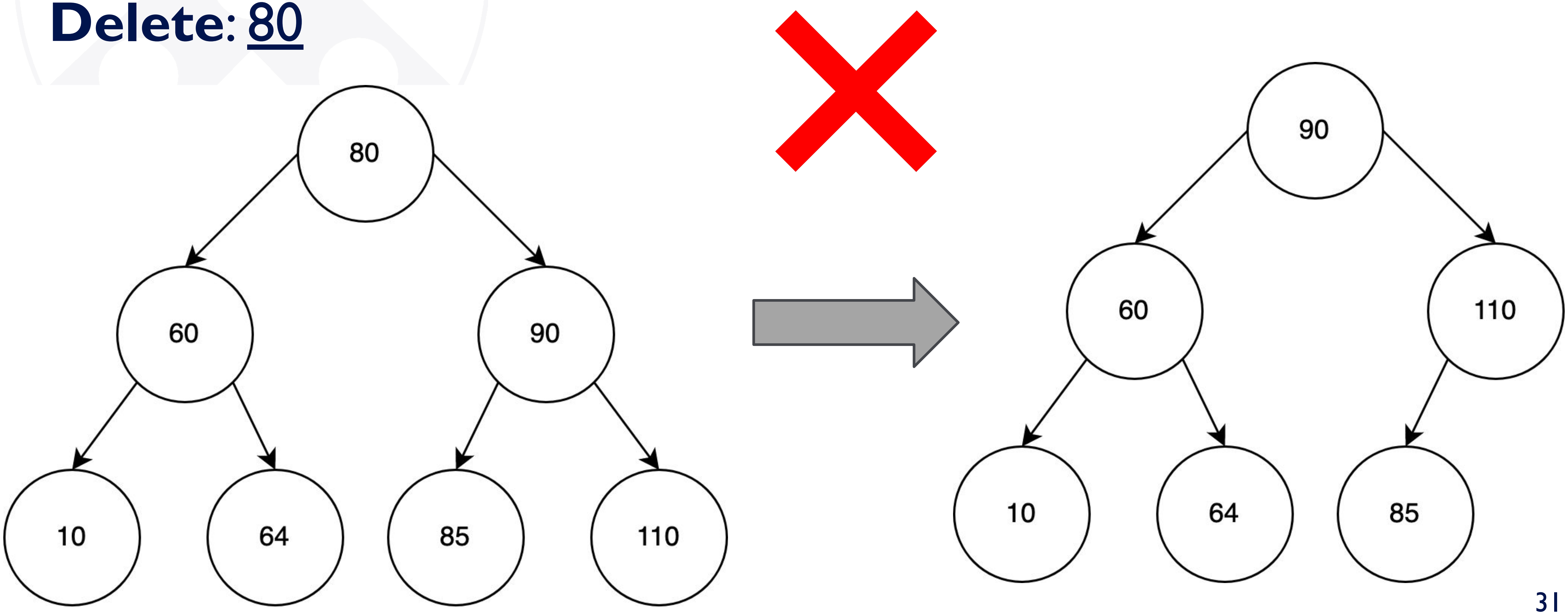
- Both subtrees are a **binary search tree**



Binary Search Trees: Deletion

Replace with 90?

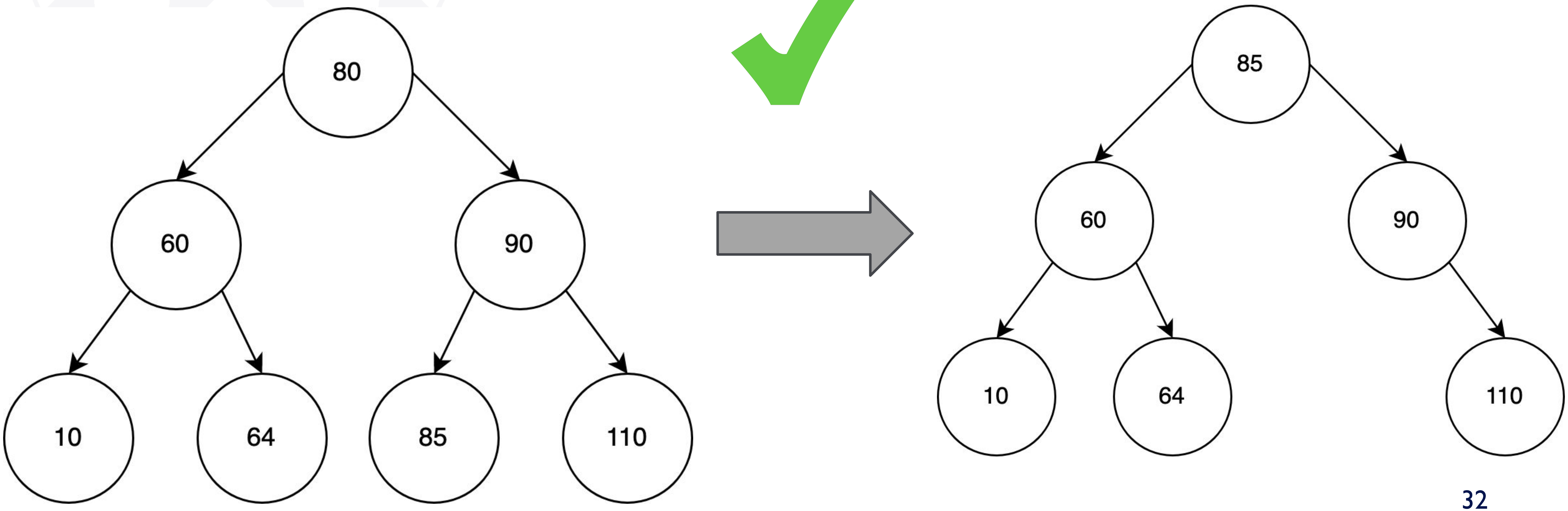
Delete: 80



Binary Search Trees: Deletion

Replace with 85?

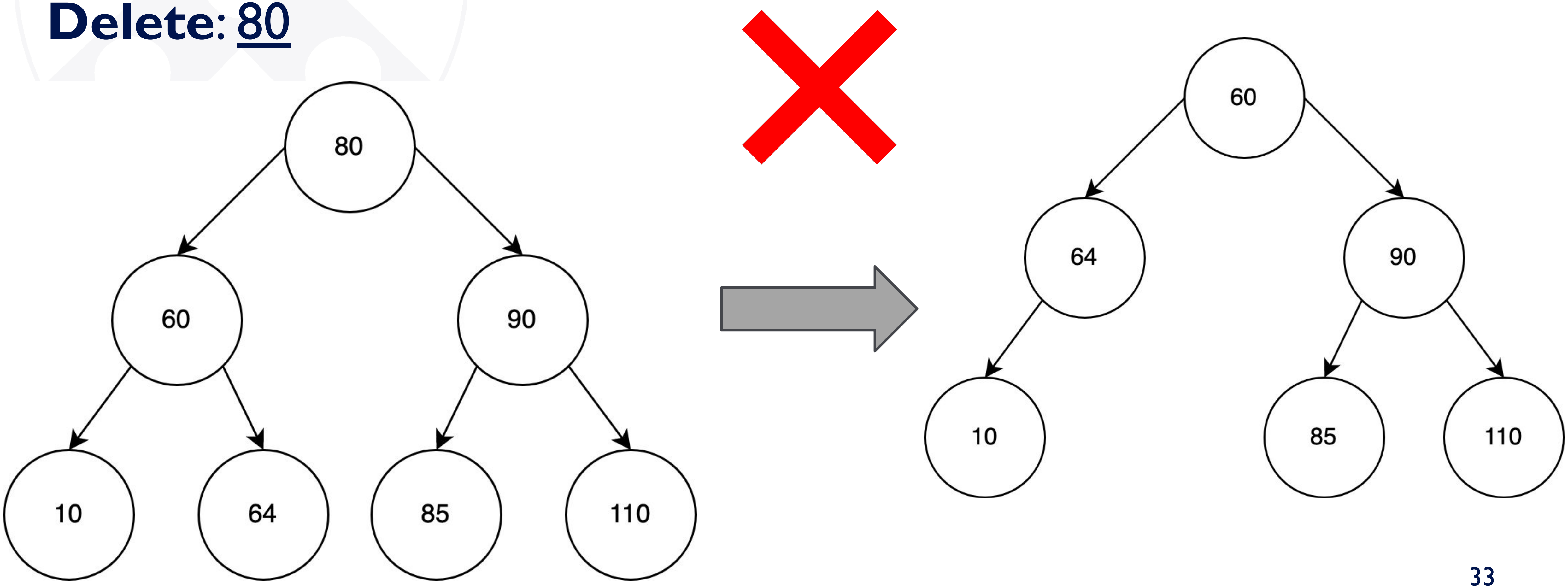
Delete: <u>80</u>



Binary Search Trees: Deletion

Replace with 60?

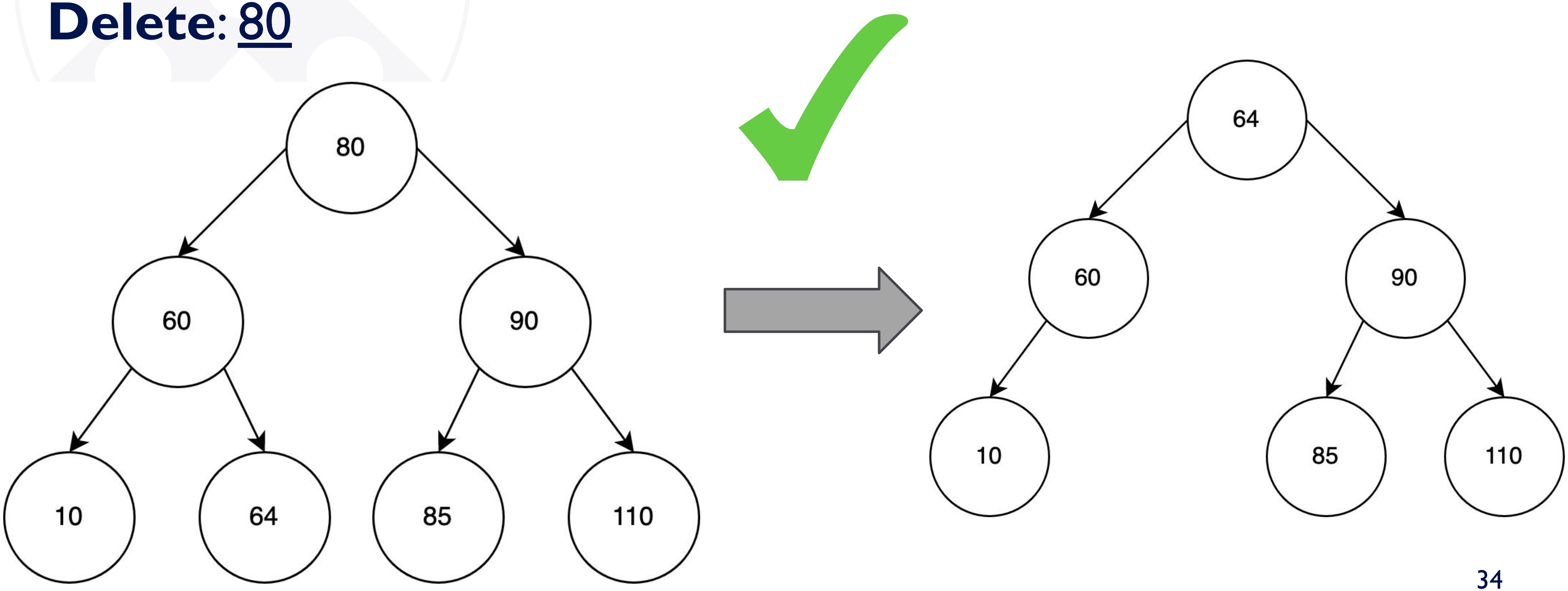
Delete: <u>80</u>



Binary Search Trees: Deletion

Replace with 64?

Delete: 80



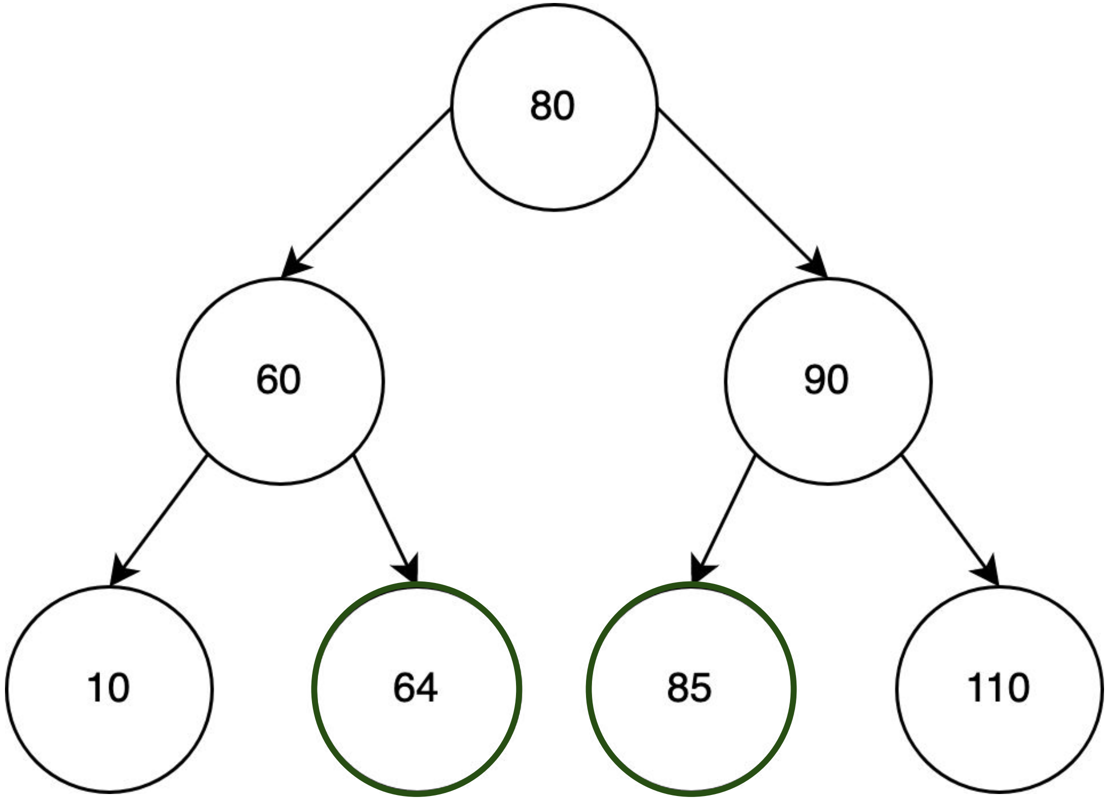
Binary Search Trees: Deletion

Deletion must maintain the properties of a BST!

Delete: <u>80</u>

Replace deleted node with either:

1. Smallest value in right subtree
2. Largest value in left subtree



Binary Search Trees: Deletion

Complexity?

Case 1: Removing a **leaf node**

O(log n)

Case 2: Removing a **node with one child**

O(log n)

Case 3: Removing a **node with two children**

O(log n)

What can go wrong?

Complexity?

Search

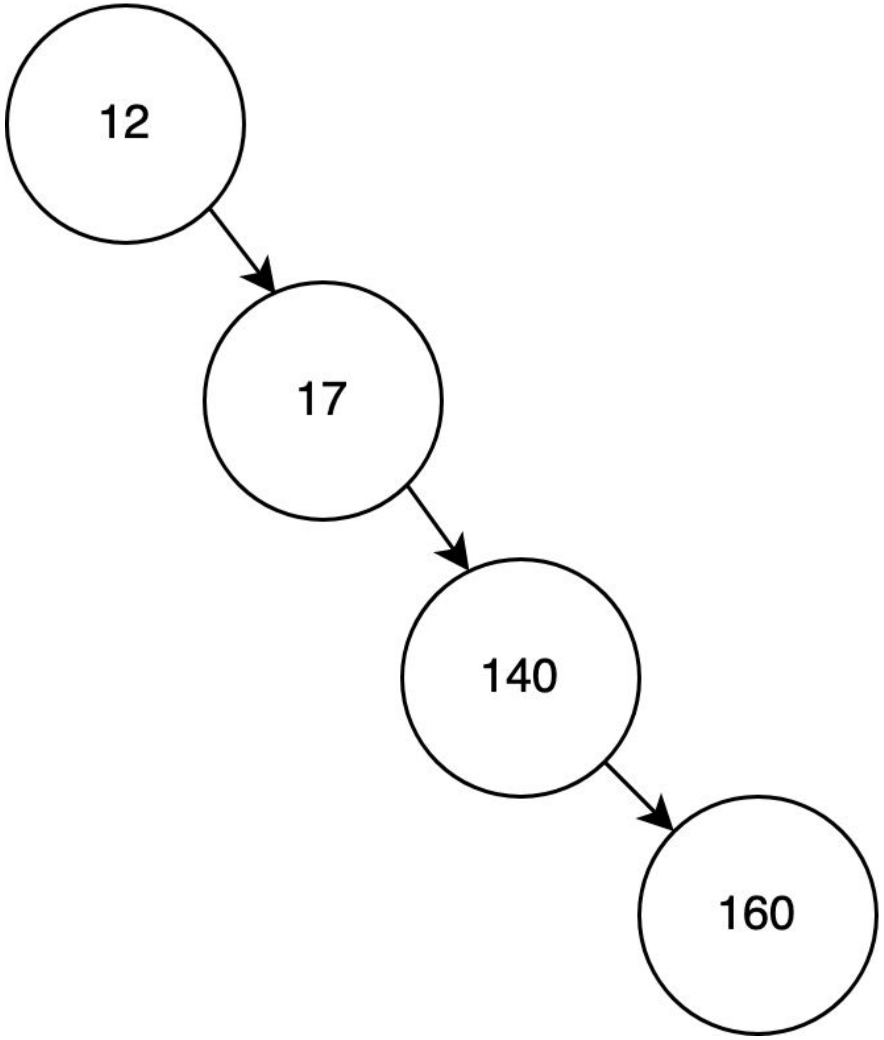
O(n)

Insertion:

O(n)

Deletion:

O(n)



Today's Lecture

1. Binary Search Trees
2. Search
3. Insertion
4. Removal
5. **Summary**

Summary

Takeaways:

Binary search trees are an efficient data structure for search

For a *balanced* binary search tree:

- Search: O(log n)
- Insertion: O(log n)
- Removal: O(log n)